PAGE
DATE

Lecture 2

acceleration can be written in 3 ways

1- $a = f(t) = \frac{dv}{dt}$

$\int dv = \int f(t)\,dt \Rightarrow v = g(t)$

2- $a = f(x) = \frac{v\,dv}{dx}$

$\int v\,dv = \int f(x)\,dx \Rightarrow \frac{1}{2}v^2 = g(x)$

3- $a = f(v)$

a) $a = \frac{dv}{dt} = f(v)$, $\int \frac{dv}{f(v)} = \int dt$

b) $a = \frac{v\,dv}{dx} = f(x)$, $\int \frac{v\,dv}{f(v)} = \int dx$

Ex 11.2 Pg 61

a) $\frac{dv}{dt} = a = -9.81$ (upwards is +ve ∴ ~~down~~ acc. is negative)

$\int_{10}^{v} dv = -9.81 \int_{0}^{t} dt$ (Final value, initial value)

$v\Big|_{10}^{v} = -9.81\,t\Big|_{0}^{t}$ $\quad v - 10 = -9.81t$ $\quad v = 10 - 9.81t$

$\frac{dy}{dt} = v = 10 - 9.81t$

$\int_{20}^{y} dy = \int_{0}^{t} (10 - 9.81t)\,dt$

$y - 20 = 10t - \frac{9.81t^2}{2}$ $\quad y = -4.95t^2 + 10t + 20$

b) y_{max} at $v = 0$, $\quad 0 = 10 - 9.81t$ $\quad t = 1.02\,s.$

$y = 25.1\,m$

c) time when ball hits ground $\Rightarrow y = 0$ $\quad -4.95t^2 + 10t + 20 = 0$

$t = 3.26\,s.$

Ex 11.3

a) $a = -kv = \frac{dv}{dt}$ $\quad \int_{v_0}^{v} \frac{dv}{v} = -\int_{0}^{t} k\,dt$ $\quad v_0$ = initial velocity

$\ln v\Big|_{v_0}^{v} = -kt$ $\quad \ln v - \ln v_0 = -kt$, $\ln \frac{v}{v_0} = -kt$

$\frac{v}{v_0} = e^{-kt}$ $\quad v = v_0 e^{-kt}$

PAGE
DATE

b) $V = \frac{dx}{dt} = V_0 e^{-kt}$ $\qquad \int_0^x dx = V_0 \int_0^t e^{-kt}\, dt$

$x = V_0 \frac{e^{-kt}}{-k}\Big|_0^t = \frac{V_0}{-k}\left(e^{-kt} - 1\right)$ $\qquad x = \frac{V_0}{-k}\left(e^{-kt} - 1\right)$

c) $a = -kV = V\frac{dv}{dx}$ $\qquad \int_{V_0}^{V} dv = -k\int_0^x dx$

$V - V_0 = -kx$, $V = V_0 - kx$

~~Integration rules~~